

Soft gluon cascades and BFKL equation

Andrei Shuvaev

*St. Petersburg Nuclear Physics Institute
188350, Gatchina, St. Petersburg district, Russia*

e-mail: shuvaev@thd.pnpi.spb.ru

Samuel Wallon

*Laboratoire de Physique Théorique¹
Université Paris XI, Centre d'Orsay, bât. 211
91405 Orsay Cedex, France*

e-mail: Samuel.Wallon@th.u-psud.fr

Abstract

In this paper we deal with high energy scattering in the Regge limit, using a soft cascade approach. We derive an evolution equation for the gluon density in soft gluons cascades in the leading logarithmic approximation of perturbative QCD, and show that this equation reproduces BFKL equation in the forward case. The whole cascade is equivalent to a single gluon whose self-energy is responsible for gluon reggeization. The same type of equation is obtained for the QED case.

¹Unité mixte 8627 du CNRS

1 Introduction

Investigation of hard scattering amplitude in the kinematics where invariant energy s is large while transferred momentum t is fixed is an important problem of QCD. The solution of this problem in the Leading Logarithmic Approximation (LLA), collecting terms of the form $(\alpha_S \ln s)^n$, was proposed about twenty five years ago, in the BFKL equation [1]. These original papers were based on an effective triple gluons vertex and a bootstrap condition in t -channel. This approach shew the relation between perturbative QCD and reggeon calculus which was proposed a decade before. The main feature of this work is the reggeization of gluon, which appears not to be elementary but composite, as being a pole in complex momentum plane, with color octet quantum number. Another important result is that Pomeron occurs as a bound state of two reggeized gluons in singlet color channel. Although BFKL equation looks like a ladder type equation, it effectively sums up an infinite number of t -channel gluons.

A few years ago a dipole picture was proposed [2, 3, 4, 5, 6, 7]. It is based on subsequent emissions of soft gluons, which iterates in the large N_c limit. In this approach elementary degrees of freedom are made of colored dipole. The equation for dipole density in this soft cascade turns out to be identical to the BFKL equation, although it was found in the multicolor limit, while original BFKL was derived for finite N_c .

The aim of this paper is to show that BFKL dynamics can be simply recovered in the forward limit only using this classical soft emission vertex used in the dipole model [8]. We deal with gluons at finite N_c rather than with dipoles at large N_c . We obtain BFKL equation as an evolution equation for the gluon density in the cascade. This equation is similar to DGLAP [9, 10, 11, 12] except that the evolution variable is $\ln 1/x$ rather than $\ln Q^2$. Some related approaches were elaborated in [13] and [14].

Our paper goes as follows. After preliminary recalling linear classical cascades properties, we generalize them in order to incorporate non abelian effects in the LLA approximation. The main idea is that despite the complicated internal structure of the cascade, the amplitude of soft emission is the same as the one off a single gluon. Using this idea we rederive BFKL equation for forward case. The same technique, when applied to QED, gives the same type of equation.

2 Classical cascade

Consider the scattering of two particles. The asymptotic behavior of the cross section is determined by the parton cascade from incoming particles or, in other words, by their wavefunction on the light cone. For large invariant energy s (the so-called Regge limit), the amplitude is dominated by t -channel gluons rather than quarks, because of their largest spin. As a consequence gluons

dominate in every channel, and quark contributions will be neglected below.

We shall consider a semi-hard region where gluons can be treated to be soft with respect to the large invariant energy s , although still perturbative. It is known that in any massless field theory scattering amplitudes are dominated by IR contributions of two types, namely collinear and soft singularities. This semi-hard region reveals soft gluons contributions, which sum up into LLA.

The physics behind soft resummation is that the field of an ultrarelativistic source can be treated as a state of almost real particles while the vertex for real or virtual soft gluon emission can be taken to be the same.

Let us denote p_A and p_B the momenta of colliding particles A and B . Neglecting their masses in the high energy limit we can take $p_A^2 = p_B^2 = 0$ so that the invariant energy reads $s = 2p_A \cdot p_B$. Vectors p_A, p_B can be used for the Sudakov decomposition of any vector as $k = \alpha p_A + \beta p_B + k_\perp$.

Each particle develops a parton cascade, so the scattering of the incoming particles A and B could be in principle reduced to elementary parton processes. However the complete description of the cascade is a very complicated problem involving all perturbative as well as nonperturbative effects. There are certain limits, where the cascade looks more simple. A well known example is deep inelastic scattering. For large virtuality Q^2 the parton density obeys the DGLAP evolution equation collecting the powers of $\alpha_S \ln Q^2$. Although predictions based on the DGLAP evolution yield good agreement with the present experimental data, this approach fails for parametrically small Bjorken variables x , when the powers of $\alpha_S \ln 1/x$ dominate. This is the region of Regge kinematics where the total invariant energy is much greater than the other invariants, including the virtuality of the deeply virtual photon. The leading $\ln 1/x$ behavior of the hard amplitude is given in this region by the BFKL theory.

An important feature of the Regge kinematics is that the partons are soft there in the sense that the main contribution appears for small longitudinal variables, $\alpha, \beta \ll 1$. Only soft emissions from the external incoming or outgoing particles should be taken into account in this approximation. Since the momenta of all incident external particles are supposed to be along p_A or p_B directions, two peculiar gauges are of special interest, namely the gauges $p_A \cdot A = 0$ and $p_B \cdot A = 0$. These gauges suppress soft emission from p_A and p_B lines respectively. In the following we shall deal with emission from p_B line in the gauge $p_A \cdot A = 0$.

The vertex for emitting a soft particle is rather universal and determined by the classical current proportional to the momentum of the source. It can be easily obtained, for example, from the triple gluon vertex in the soft limit. Consider emission off the incoming particle B , as illustrated in Fig. 1.

This incoming line is attached to an amplitude (shown as a blob in Fig. 1) whose peculiar form plays no role so long as we are dealing with soft emission.

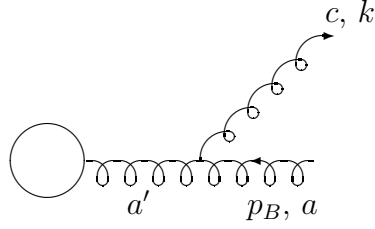


Figure 1: Soft vertex for incoming line. The blob symbolizes any amplitude connected to the incoming line.

The soft vertex reads

$$\Gamma_c^\lambda(k) = g \frac{p_B \cdot \varepsilon^\lambda(k)}{p_B k - i\delta} T_c = g \frac{p_B \cdot \varepsilon^\lambda(k)}{\alpha_2^{\frac{s}{2}} - i\delta} T_c, \quad (1)$$

where $\varepsilon^\lambda(k)$ is the helicity vector of the soft emitted or absorbed gluon carrying momentum k and color index c . The matrix T^c depends on the representation of the color group. For gluon like object it is expressed through the structure constants, $T_{a'a}^c = if_{a'ca}$, the indices c, a', a being in the adjoint representation.

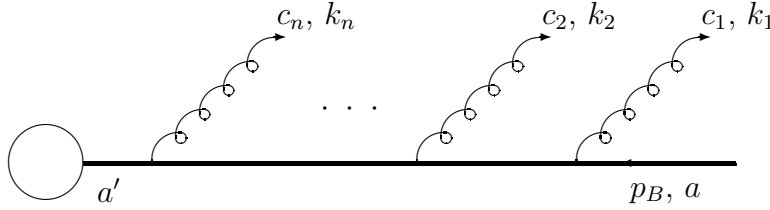


Figure 2: Amplitude for n gluon emission or absorption off an incoming gluon-like object, represented as two thin lines.

The amplitude for emission or absorption of n gluons (see Fig. 2) is given by the product of the elementary vertices,

$$\begin{aligned} \Gamma_{c_n, \dots, c_1}^{\mu_n, \dots, \mu_1}(k_1, \dots, k_n) A_{\mu_n, c_n}(k_n) \cdots A_{\mu_1, c_1}(k_1) &= (T^{c_n} \cdots T^{c_1})_{a'a} \times \\ \times g \frac{p_B \cdot A_{c_n}(k_n)}{(\alpha_1 + \alpha_2 + \dots + \alpha_n)^{\frac{s}{2}} - i\delta} \cdots g \frac{p_B \cdot A_{c_2}(k_2)}{(\alpha_1 + \alpha_2)^{\frac{s}{2}} - i\delta} g \frac{p_B \cdot A_{c_1}(k_1)}{\alpha_1^{\frac{s}{2}} - i\delta}. \end{aligned} \quad (2)$$

Here the field A is the asymptotic free field

$$A_{\mu, c}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3 k}{2k_0} \left[e^{ikx} \varepsilon_\mu^\lambda(k) a_{\lambda, c}^+(k) + e^{-ikx} \varepsilon_\mu^\lambda(k) a_{\lambda, c}(k) \right],$$

whose positive and negative frequency part corresponds respectively to emission and absorption of gluons. Using the relation

$$\frac{1}{\alpha_1^{\frac{s}{2}} - i\delta} = i \int_{-\infty}^{\infty} dz e^{i\alpha_1^{\frac{s}{2}} z} \theta(-z)$$

we get

$$\begin{aligned} & \int d^4 k_1 \cdots d^4 k_n \Gamma_{c_n, \dots, c_1}^{\mu_n, \dots, \mu_1}(k_1, \dots, k_n) A_{\mu_n, c_n}(k_n) \cdots A_{\mu_1, c_1}(k_1) \\ &= \frac{(ig)^n}{n!} \int dz_1 \cdots dz_n P[n_B \cdot A(z_n n_B) \cdots n_B \cdot A(z_1 n_B)]_{a'a}, \end{aligned}$$

with

$$A_\mu(x) = \int d^4 k e^{ikx} A_{\mu, c}(k) T^c$$

and where the symbol P means the conventional ordering of the fields. The sum over the arbitrary number of soft gluons is given by a P-exponent along the momentum direction of the parent particle:

$$\begin{aligned} & \sum_n \int d^4 k_1 \cdots d^4 k_n \Gamma_{c_n, \dots, c_1}^{\mu_n, \dots, \mu_1}(k_1, \dots, k_n) A_{\mu_n, c_n}(k_n) \cdots A_{\mu_1, c_1}(k_1) \\ &= P \exp \left[ig \int_{-\infty}^0 dz n_B \cdot A(z n_B) \right]. \end{aligned} \quad (3)$$

A similar computation for outgoing line results in replacing in the previous formula $\int_{-\infty}^0$ with \int_0^∞ . These formulae are nothing more than the Wilson line taken on the trajectory along the momentum of the incoming (resp. outgoing) particle.

3 Inclusion of interaction among emitted particles

The previous computation only takes into account emissions from the parent line which is the incoming line (resp. outgoing). The above formulae are strictly speaking only valid for QED. They do not take into account the interaction between emitted particles. In QCD, in the soft approximation, these corrections correspond to subsequent decays of the emitted gluons. Indeed in this approximation only end lines emissions contribute, and thus to get the total cascade tree-level amplitude each field in the P-exponent has to be replaced by the P-exponent itself. Generally this turns into a complicated non-linear equation. Another problem is to incorporate loop corrections. This second problem will be considered in section 4.

The first problem, the partons subsequent decays, simplifies in the Regge kinematics implying the longitudinal momenta to be small in the formula (2), $\alpha, \beta \ll 1$, while

$$\alpha s, \beta s \gg k_\perp^2, \quad \alpha \beta s \sim k_\perp^2, \quad (4)$$

where k_\perp is a typical transverse momentum scale. This kinematics ensures the soft vertex to be of the form (1) regardless from where a particular gluon is emitted off or where it is absorbed in. Indeed, the emission of a gluon with

momentum $k_2 = \alpha p_A + \beta p_B + k_{2\perp}$ off the parent gluon carrying momentum $k_1 = a p_A + b p_B + k_{1\perp}$ is again given by a soft vertex similar to (1)

$$\Gamma_c^\lambda(k_2) = g \frac{k_1 \cdot \varepsilon^\lambda(k_2)}{k_1 \cdot k_2} T_c. \quad (5)$$

In the light cone gauge $p_A \cdot A(x) = 0$, the polarization vector reads

$$\varepsilon_\mu^\lambda(k) = \varepsilon_A^\lambda(k) \cdot p_{A\mu} + \varepsilon_{\perp\mu}^\lambda(k) \quad (6)$$

with

$$\sum_\mu \varepsilon_{\perp\mu}^\lambda(k) \cdot \varepsilon_{\perp\mu}^{\lambda'}(k) = \delta^{\lambda\lambda'}.$$

The emitted gluons are on-shell. Anyway, the gluons which dominate the amplitude in the Regge limit are soft and thus quasi-real, therefore the on-shell condition is unessential. The transversality condition for these quasi-real gluons reads $k \cdot \varepsilon = 0$, which implies for the helicity vector (6)

$$\varepsilon_A^\lambda(k) = 2 \frac{k_\perp \cdot \varepsilon_\perp^\lambda(k)}{\beta s}. \quad (7)$$

In this soft gluon approximation, $\beta \ll b$ and assuming the typical transverse momenta to be of the same order, $k_{1\perp}^2 \sim k_{2\perp}^2$, the numerator of the soft vertex is approximated as

$$k_1 \cdot \varepsilon^\lambda(k_2) = b \varepsilon_A^\lambda \frac{s}{2} - k_{1\perp} \varepsilon_\perp^\lambda \approx b p_B \cdot \varepsilon^\lambda(k_2).$$

Using the mass shell conditions, $a = k_{1\perp}^2 / bs$, $\alpha = k_{2\perp}^2 / \beta s$, the denominator also simplifies as

$$k_1 \cdot k_2 = (a \beta + b \alpha) \frac{s}{2} - k_{1\perp} \cdot k_{2\perp} \approx b \alpha \frac{s}{2}.$$

It follows that the vertex (5) can be written as

$$\Gamma_c^\lambda(k_2) = g \frac{p_B \cdot \varepsilon^\lambda(k_2)}{p_B \cdot k_2} T_c, \quad (8)$$

which has exactly the universal form (1). It confirms the physical picture that soft emission is determined by the total current of the source rather than by its internal structure. The vector p_B is the momentum of the source.

With this universal vertex combined with Jacobi identity for the color matrices T^c the emission of the new soft particle from the "ends" a and c in the Fig. 1 looks like if it were effectively emitted off the line a' at the left from the particle c . This is illustrated in Fig. 3.

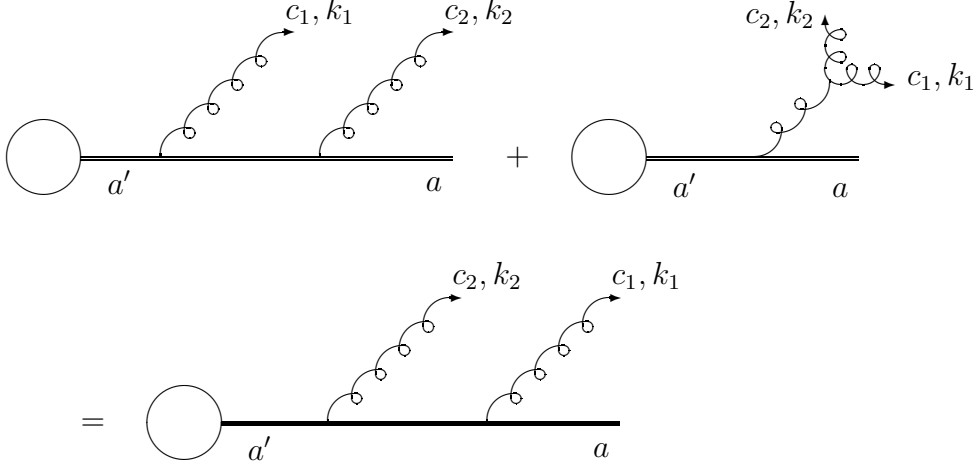


Figure 3: Illustration of the soft gluon universality using Jacobi identity.

Since the emitted gluon is real, $k^2 = \alpha \beta s - k_\perp^2 = 0$, and thus, combining Eqs.(6) and (7), the vertex (8) can be rewritten in the form

$$\Gamma_c^\lambda(k) = g T^c \frac{2k_\perp \cdot \epsilon_\perp}{k_\perp^2}. \quad (9)$$

Repeating the new emissions from all "ends" in the same manner we come back to the Fig. 2, where now the gluons are ordered according to their β value, the smaller β being on the left of the largest ones.

Note that the decreasing order of β variables,

$$\beta_1 \gg \beta_2 \gg \cdots \gg \beta_n, \quad (10)$$

implies in the Regge kinematics increasing order of α 's,

$$\alpha_1 \ll \alpha_2 \ll \cdots \ll \alpha_n,$$

so that $\alpha_1 + \alpha_2 \simeq \alpha_2$, $\alpha_1 + \cdots + \alpha_n \simeq \alpha_n$. This property allows to present the resulting soft cascade tree amplitude through an ordered exponent in momentum space,

$$\Phi_{tree} = P_\beta \exp \left\{ \int_{x_\lambda}^1 d\beta d^2 k_\perp V(\beta, k_\perp) \right\},$$

with

$$V(\beta, k_\perp) = V^+(\beta, k_\perp) + V^-(\beta, k_\perp) \quad (11)$$

and

$$V^\pm(\beta, k_\perp) = \frac{g}{(2\pi)^{\frac{3}{2}}} \frac{1}{2\beta} \frac{p_B \cdot \epsilon^\lambda(\beta, k_\perp)}{p_B \cdot k} a_{\lambda,c}^\pm(\beta, k_\perp) T^c. \quad (12)$$

In the gauge $p_A \cdot A = 0$, this equation reduces to

$$V^\pm(\beta, k_\perp) = \frac{2gT^c}{(2\pi)^{\frac{3}{2}}} \frac{1}{2\beta} \frac{k_\perp \cdot \epsilon_\perp^\lambda(\beta, k_\perp)}{k_\perp^2} a_{\lambda,c}^\pm(\beta, k_\perp) \quad (13)$$

where we have used expression (9). This expression incorporates both emission and absorption of the partons described by the light cone creation, $a_{\lambda c}^+(\beta, q_\perp)$, and annihilation, $a_{\lambda c}(\beta, q_\perp) \equiv a_{\lambda c}^-(\beta, q_\perp) = a_{\lambda c}^+(-\beta, -q_\perp)$, operators labelled by the longitudinal momentum fraction $\beta \geq 0$, transverse momentum q_\perp , polarization and color indices λ and c , which satisfies

$$[a_{\lambda c}(\beta, q_\perp), a_{\lambda' c'}^+(\beta', q'_\perp)] = 2\beta \delta_{\lambda\lambda'} \delta_{cc'} \delta(\beta - \beta') \delta^{(2)}(q_\perp - q'_\perp).$$

The ordering symbol P_β means that the fields $A(\beta)$ with the smallest β value are on the left from the fields with the largest ones. The minimal value $\beta = x_\lambda$ plays the role of an infrared cut-off in the amplitude Φ_{tree} . Note that in the case of incoming line, the variable “time” z in the P-exponent (3) is Fourier conjugated to the longitudinal Sudakov variable α , and thus ordered oppositely. Since on the other hand, for mass-shell particles $\alpha \sim 1/\beta$, it turns out that $|z| \sim \beta$. Thus, P-exponent defined with respect to z is in accordance with P_β .

When the cut-off value is lowered, that is when we take $x'_\lambda < x_\lambda$, it allows for the emission or absorption of an extra soft particle whose longitudinal momentum lies in the interval $x'_\lambda < \beta < x_\lambda$. This yields the new amplitude

$$\Phi'_{tree} = \left[1 + \int_{x_\lambda - \delta x_\lambda}^{x_\lambda} d\beta \int d^2 k_\perp V(\beta, k_\perp) \right] \Phi_{tree}, \quad (14)$$

the amplitude Φ_{tree} standing as the source for the new soft particles. The whole tree amplitude can be symbolically presented as an infinite product of elementary emissions or absorptions in the infinitesimal intervals Δx ,

$$\Phi_{tree} = \prod_{x_\lambda}^1 \left[1 + \int_{x - \Delta x}^x d\beta \int d^2 k_\perp V(\beta, k_\perp) \right]. \quad (15)$$

4 Virtual corrections and evolution of the cascade wave function

The previous form is convenient to include the virtual contributions. Besides the amplitude to emit or absorb a real gluon at each elementary step there is an amplitude for the case when the new phase volume in the longitudinal space remains non-occupied. In other words it means that the emitted gluon is reabsorbed at the same step, which corresponds to a loop correction. It does not change the number of particles, but it is responsible for the renormalization of the whole state.

Despite the composite internal structure of the whole system of cascade and source, within the soft emission approach this system looks like the source carrying given momentum and color. The virtual correction results into self-energy insertion into the source propagator. In particular, if the source is gluon like, then the full system looks like a gluon. In this case, the virtual correction

results into self-energy insertion in the gluon propagator. In LLA and for the cascade plus source system of momentum $q \simeq p_B + q_\perp$, the propagator of this system is modified as

$$D_{\mu\nu}(q) = \frac{1}{q^2} \left[\delta^{\mu\nu} - \frac{q^\mu p_A^\nu + q^\nu p_A^\mu}{p_A \cdot q} \right] \frac{1}{1 - \pi(x_\lambda, q_\perp^2)},$$

with the function $\pi(x_\lambda, q_\perp^2)$ determining z -factor, or normalization of the state. To keep the value of the norm fixed, the cascade plus source wavefunction is multiplied by $z_{[x_\lambda, 1]}^{-1/2}(q_\perp)$, where q_\perp is the total transverse momentum of the system whose constituents fill the interval of longitudinal momenta $[x_\lambda, 1]$,

$$z_{[x_\lambda, 1]}(q_\perp) = \frac{1}{1 - \pi(x_\lambda, q_\perp^2)}.$$

The contribution of the soft gluons appearing at the next evolution step in the interval $[x_\lambda - \delta x_\lambda, x_\lambda]$ should be compensated by the factor $z_{[x_\lambda - \delta x_\lambda, x_\lambda]}^{-1/2}(q_\perp)$. Basically for a finite interval one can write a loop expansion of the form

$$z_{[x_\lambda - \delta x_\lambda, x_\lambda]}(q_\perp) = 1 + \sum_{n \geq 1} z_n(q_\perp) \ln^n \frac{x_\lambda}{x_\lambda - \delta x_\lambda},$$

only the first variation determining the evolution equation,

$$z_{[x_\lambda - \delta x_\lambda, x_\lambda]}^{-1/2}(q_\perp) = 1 - \omega(q_\perp) \frac{\delta x_\lambda}{x_\lambda} + O\left(\frac{\delta x_\lambda}{x_\lambda}\right)^2. \quad (16)$$

In LLA function $\omega(q_\perp)$ is given by a one loop diagram calculated in the axial gauge.

Before discussing an explicit form of the virtual contribution note that the momentum q_\perp in the argument of ω is the total transverse momentum of the system of cascade and source, which emit a soft particle as a whole. Introducing the total momentum operator \hat{P}_\perp , $\omega(\hat{P}_\perp)$ obviously gives $\omega(q_\perp)$ when acting on a whole system of cascade plus source state of total momentum q_\perp . Thus, in a similar way as in the tree case (see Eq.(14)), one step of evolution reads

$$\Phi(x_\lambda - \delta x_\lambda) = \left[1 - \omega(\hat{P}_\perp) \frac{\delta x_\lambda}{x} + \int_{x - \delta x_\lambda}^x d\beta \int d^2 k_\perp V(\beta, k_\perp) \right] \Phi(x_\lambda).$$

The full cascade S matrix can be symbolically written through the elementary steps product¹

$$\Phi(x_\lambda) = \prod_{x_\lambda}^1 \left[1 - \omega(\hat{P}_\perp) \frac{\Delta x}{x} + \int_{x - \Delta x}^x d\beta \int d^2 k_\perp V(\beta, k_\perp) \right], \quad (17)$$

¹Here δx_λ denotes the variation of the cut-off value while Δx is taken as a notation for an infinitesimal step in the infinite product representation of the cascade S matrix. Principally one can put $\delta x_\lambda = \Delta x$.

where the brackets are ordered from the smallest β 's values at the left to the greatest at the right. The operator

$$\Phi^+(x_\lambda) = \prod_{x_\lambda}^1 \left[1 - \omega(\hat{P}_\perp) \frac{\Delta x}{x} + \int_{x-\Delta x}^x d\beta \int d^2 k_\perp V^+(\beta, k_\perp) \right]$$

acting on the vacuum creates the soft constituents of the cascade, while the operator

$$\Phi^+(x_\lambda, p_\perp) = \frac{1}{(2\pi)^2} \int d^2 z e^{-ip_\perp z} e^{i\hat{P}z} \Phi^+(x_\lambda) e^{-i\hat{P}z}$$

picks out the components with a fixed transverse momentum. Let us introduce the operator $b_{\sigma,a}^+(p_B, q_\perp)$ for creating the bare source, having transverse momentum q_\perp , helicity and color indices σ, a , and denote this state as $|p_B, q_\perp, a, \sigma\rangle_s = b_{\sigma,a}^+(p_B, q_\perp)|0\rangle$. Then the state

$$|p_B, x_\lambda, q_\perp, a, \sigma\rangle = \int d^2 p_\perp \Phi^+(x_\lambda, p_\perp) b_{\sigma,a}^+(p_B, q_\perp - p_\perp) |0\rangle$$

describes the system of cascade plus source with given total transverse momentum q_\perp , cut-off value x_λ , and source helicity and color. The total transverse momentum of the cascade is given by p_\perp . The operator \hat{P}_\perp acting on the right results at every step into the transverse momentum of the total system at the previous cut-off value.

Changing x_λ we evidently have for the variation of the cascade plus source state

$$\begin{aligned} \delta |p_B, x_\lambda, q_\perp, a, \sigma\rangle &= -\frac{\delta x_\lambda}{x_\lambda} \omega(q_\perp) |p_B, q_\perp, x_\lambda, a, \sigma\rangle \\ &+ \int_{x_\lambda - \delta x_\lambda}^{x_\lambda} d\beta \int d^2 k_\perp V^+(\beta, k_\perp) |p_B, (q - k)_\perp, x_\lambda, a, \sigma\rangle. \end{aligned} \quad (18)$$

Let Q be an operator, which probes the parton distribution in the cascade, for instance,²

$$Q = \int_{x_Q}^1 \frac{dx}{2x} d^2 l_\perp f(x, l_\perp) a_{\sigma c}^+(x, l_\perp) a_{\sigma c}(x, l_\perp) \quad (19)$$

with a weight function $f(x, l_\perp)$. The average value is generally expressed through the density function n_f , depending on the cascade total transverse momentum and cut-off,

$$\langle p_B, x_\lambda, q_\perp, a, \sigma | Q | p_B, x_\lambda, q'_\perp, a', \sigma' \rangle = \delta_{a,a'} \delta_{\sigma,\sigma'} \delta^{(2)}(q_\perp - q'_\perp) n_f(x_\lambda, q_\perp).$$

²In principle the operator Q could probe both the cascade content and the source, without modifying the following discussion. One could add to Q a term

$$1/2 \int d^2 l_\perp f'(l_\perp) b_{\sigma,c}^+(p_B, l_\perp) b_{\sigma,c}(p_B, l_\perp)$$

acting on the source. If the source is gluon-like this is the same as to include in the weight function $f(x, l_\perp)$ in Eq.(19) a term proportional to $\delta(1-x)$.

Its evolution with x_λ value is determined by the variation of the state (18). If we suppose that the operator Q has a natural cut-off $x_Q \gg x_\lambda$, then the new emitted gluon does not affect the operator vertex (the operators $a_{\lambda,c}(\beta, k_\perp)$ commute with Q for $x_\lambda - \delta x_\lambda < \beta < x_\lambda$) and the variation of the matrix element decays into the sum

$$\Delta n_f = \Delta_1 n_f + \Delta_2 n_f,$$

as illustrated in Fig. 4. The first term is given by the extra particle amplitude

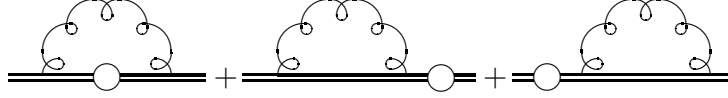


Figure 4: Variation of the density with respect to the cut-off, in the case $x_Q \gg x_\lambda$. The double thick line stands for the system of source plus gluon cascade.

squared times the average of the operator Q over the rest part of the amplitude corresponding to the previous cut-off value x_λ and recoil transverse momentum. It corresponds to the first diagram in Fig. 4 and can be written symbolically

$$\Delta_1 n_f(x_\lambda, q_\perp) = \langle p_B, x_\lambda, q_\perp - k_\perp, a, \sigma | a_{\lambda,c} Q a_{\lambda,c}^+ | p_B, x_\lambda, q'_\perp - k_\perp, a', \sigma' \rangle \quad (20)$$

or in explicit form

$$\Delta_1 n_f(x_\lambda, q_\perp) = 2N_c \frac{g^2}{(2\pi)^3} \ln \frac{x_\lambda}{x'_\lambda} \int \frac{d^2 k_\perp}{k_\perp^2} n_f(x_\lambda, q_\perp - k_\perp). \quad (21)$$

Note that the previous equation is written in the case where the source is in the adjoint representation. In the general case, N_c should be replaced by the Casimir of the corresponding representation.

It is important to note that in soft loops calculations gluons can be considered as real, in accordance with the fact that there is almost no difference between real and virtual massless soft particles. Indeed, consider the loop where one single gluon is emitted and reabsorbed, as illustrated in the Fig. 4. The α integral which occur in the loops can be closed around the pole of the emitted gluon propagator. For, using the same technique which leads to the eikonal type expression (1), the denominator of the integrand reads $(\alpha - i\delta)^2(\alpha\beta s - k_\perp^2 + i\delta)$ for the first graph of Fig. 4 or $(\alpha - i\delta)(\alpha\beta s - k_\perp^2 + i\delta)$ for the second and third graphs. Both denominators leave the singularities in α plane on the opposite side of the real axis. The numerator of the gluon propagator in light-cone gauge reads

$$d_{\mu\nu}(k) = -\varepsilon_\mu^\lambda(k) \varepsilon_\nu^\lambda(k) - \frac{4k^2}{\beta^2 s^2} p_{A\mu} p_{A\nu}.$$

When performing the α integral, if among the two poles one choose to close around the physical pole of the gluon, the second term drops out since it cancels the α singularity in the propagator. Thus, only the first term remains. One then immediately gets the soft universal vertex squared (9) for the first graph.

The second term in the variation arises due to virtual correction,

$$\Delta_2 n_f(x_\lambda, q_\perp) = -2 \ln \frac{x_\lambda}{x'_\lambda} \omega(q_\perp). \quad (22)$$

It is illustrated by the second and third diagrams of Fig. 4.

Differentiating with respect x_λ we arrive at the evolution equation

$$x_\lambda \frac{\partial}{\partial x_\lambda} n_f(x_\lambda, q_\perp) = -2 N_c \frac{g^2}{(2\pi)^3} \int \frac{d^2 k_\perp}{(q - k)_\perp^2} n_f(x_\lambda, k_\perp) - 2\omega(q_\perp) n_f(x_\lambda, q_\perp).$$

In the case where x_Q is smaller than the typical value of x_λ , there appears an additional term when the extra soft gluon operator is contracted with the operator Q , as shown by the additional fourth diagram in Fig. 5. It gives the

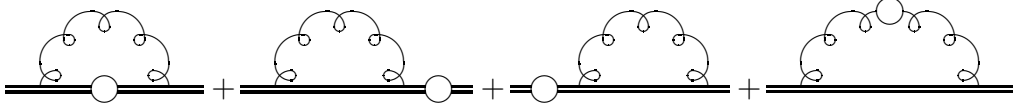


Figure 5: Variation of the density with respect to the cut-off, in the case $x_Q \ll x_\lambda$.

following contribution to the variation of n_f :

$$\Delta_3 n_f(x_\lambda, q_\perp) = 2 N_c \frac{g^2}{(2\pi)^3} f(x_\lambda, q_\perp) \ln \frac{x_\lambda}{x'_\lambda}. \quad (23)$$

The full inhomogeneous equation thus reads

$$x_\lambda \frac{\partial}{\partial x_\lambda} n_f(x_\lambda, q_\perp) = 2 N_c \frac{g^2}{(2\pi)^3} \int \frac{d^2 l_\perp}{l_\perp^2} \ln \frac{x_\lambda}{x'_\lambda} - 2 N_c \frac{g^2}{(2\pi)^3} \int \frac{d^2 k_\perp}{(q - k)_\perp^2} n_f(x_\lambda, k_\perp) - 2\omega(q_\perp) n_f(x_\lambda, q_\perp). \quad (24)$$

If one is interested in the density number of gluon of momentum r_\perp , the weight function f in Eq.(19) should be chosen as

$$f(x, l_\perp) = \frac{1}{2} \delta^2(l_\perp - r_\perp), \quad (25)$$

the 1/2 factor being related to the average over gluon transverse polarization.

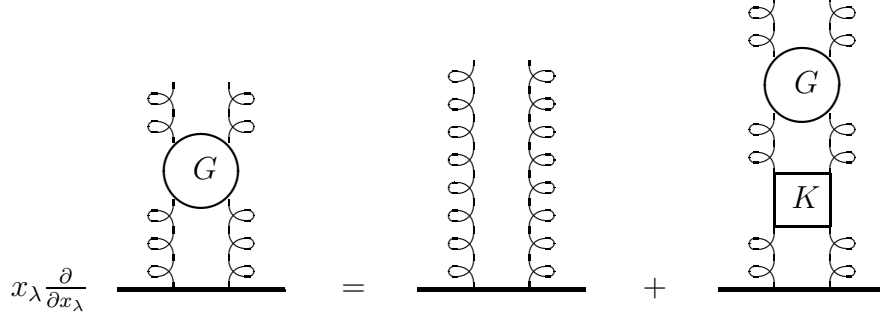


Figure 6: BFKL equation for the gluon density in a cascade. The double line stands for the source.

In that case, Eq.(24) turns into the BFKL equation (for $t = 0$)

$$x_\lambda \frac{\partial}{\partial x_\lambda} G(x_\lambda, r_\perp, q_\perp) = -N_c \frac{g^2}{(2\pi)^3} \frac{\delta^2(q_\perp - r_\perp)}{r_\perp^2} - 2N_c \frac{g^2}{(2\pi)^3} \int \frac{d^2 k_\perp}{(q - k)_\perp^2} G(x_\lambda, r_\perp, k_\perp) - 2\omega(q_\perp) G(x_\lambda, r_\perp, q_\perp). \quad (26)$$

The inhomogeneous term, which is of lowest order, can be interpreted as the initial gluon contribution in the absence of cascade. Since we consider here gluons emitted from a source, it contains a factor $N_c g^2/(2\pi)^3$, multiplying $1/r_\perp^2$ which is reminiscent of the two t -channel gluon propagators (one being compensated by polarization contribution after angular averaging of k_\perp).

The equation Eq.(26) looks like a Bethe-Salpeter equation with a kernel K acting from below, which is illustrated in Fig. 6.

Actually the density G is only a function of $p_\perp - r_\perp$ due to translational invariance. It means that Eq.(26) could equally be written as a Bethe-Salpeter equation with a kernel acting from above, by just an exchange between of variables r_\perp and p_\perp .

A natural physical value for the IR cut-off x_λ is $x_\lambda \sim \mu^2/s$ where μ^2 is some typical scale for transverse momentum. This follows from the Regge kinematics (4).

Using Eqs.(19) and (25), one can finally obtain the unintegrated parton density from G as

$$g(x_\lambda, r_\perp, q_\perp) = \frac{1}{2x_\lambda} \langle p_B, x_\lambda, q_\perp, a, \sigma | \frac{1}{2} \sum_{\lambda, c} a_{\sigma, c}^+ a_{\sigma, c} | p_B, x_\lambda, q'_\perp, a', \sigma' \rangle = -\frac{\partial}{\partial x_\lambda} G(x_\lambda, r_\perp, q_\perp). \quad (27)$$

In this formula, r_\perp is the transverse momentum at which parton density is measured while q_\perp is the total transverse momentum of the cascade plus source.

5 Computation of ω

5.1 Evaluation of ω based on the gluon cascade universality

The expression (21) provides a simple way to find the function $\omega(q_\perp)$ based on universality. From the point of view of emitted gluon, the whole system corresponding to the previous cut-off value plays the role of the source, and the z -factor is prescribed to this whole system. On the other hand, this whole system looks like a gluon (in the case where bare source is gluon like). Thus, the z -factor for the whole system and for a single gluon should have the same functional form. What we need is the virtual correction for the whole system, but we shall calculate instead the z -factor of the dressed emitted gluon. The inclusion of z -factor corresponds to the replacement $a_{\lambda,\sigma} \rightarrow z^{1/2} a_{R\lambda,\sigma}$ in the expression (20). With z -factor included the formula (21) reads

$$\begin{aligned} \Delta_1 n_f(x_\lambda, q_\perp) &= 2N_c \frac{g^2}{(2\pi)^3} \ln \frac{x_\lambda}{x_\lambda - \delta x_\lambda} \\ &\times \int \frac{d^2 p_\perp}{p_\perp^2} z_{[x_\lambda - \delta x_\lambda, x_\lambda]}(p_\perp) n_f(x_\lambda, q_\perp - p_\perp). \end{aligned} \quad (28)$$

On the other hand there is a dual description of the same dressed gluon as a composite cascade state spread over the interval $[x_\lambda - \delta x_\lambda, x_\lambda]$ and carrying transverse momentum p_\perp . The two particles component of this state is given by the amplitude to emit two soft gluons off the source (which is the whole system corresponding to the previous cut-off value),

$$\begin{aligned} |p_B, q_\perp, x_\lambda - \delta x_\lambda, a\sigma\rangle_2 &= igf_{ac_1d} igf_{dc_2b} \int_{x_\lambda - \delta x_\lambda}^{x_\lambda} \frac{dx}{2x} \int_{x_\lambda - \delta x_\lambda}^x \frac{d\beta}{2\beta} \int \frac{d^2 k_\perp}{(2\pi)^{3/2}} 2 \frac{k_\perp \cdot \varepsilon_{\sigma_2}}{k_\perp^2} \\ &\times \frac{1}{(2\pi)^{3/2}} 2 \frac{(p-k)_\perp \cdot \varepsilon_{\sigma_1}}{(p-k)_\perp^2} a_{\sigma_2 c_2}^+(\beta, k_\perp) a_{\sigma_1 c_1}^+(x, p_\perp - k_\perp) |p_B, q_\perp - p_\perp, x_\lambda, b, \sigma\rangle, \end{aligned}$$

the gluons being ordered with respect to their longitudinal momenta,

$$x_\lambda - \delta x_\lambda < \beta < x < x_\lambda,$$

which is reflected in the integration limits.

Averaging the operator with this amplitude we arrive at the following expression for the gluon density variation

$$\begin{aligned} \Delta_1 n_f(x_\lambda, q_\perp) &= 2N_c \frac{g^2}{(2\pi)^3} \frac{1}{2} \ln^2 \frac{x_\lambda}{x_\lambda - \delta x_\lambda} \int \frac{d^2 p_\perp}{p_\perp^2} n_f(x_\lambda, q_\perp - p_\perp) \\ &\times 2N_c \frac{g^2}{(2\pi)^3} \int d^2 k_\perp \frac{p_\perp^2}{k_\perp^2 (p-k)_\perp^2}. \end{aligned}$$

Note that the double integration in x and β results into $1/2 \ln^2 \frac{x_\lambda}{x_\lambda - \delta x_\lambda}$. This factor $1/2$ reflects the Bose symmetry of the two gluons system. Recalling Eq.(16) and Eq.(28) we get

$$2\omega(p_\perp) = N_c \frac{g^2}{(2\pi)^3} \int d^2 k_\perp \frac{p_\perp^2}{k_\perp^2 (p-k)_\perp^2}. \quad (29)$$

5.2 Direct calculation

The formula (29) can be compared with a direct calculation of the polarization operator. The exact one loop result in the dimensional regularization ($D = 2 + 2\epsilon$) and axial gauge is

$$\pi(p_\perp) = -2N_c \frac{g^2}{8\pi^2} \Gamma\left(1 - \frac{D}{2}\right) \int_0^1 d\beta [\beta(1-\beta) p_\perp^2/4\pi]^{\frac{D}{2}-1} (1-\beta) K(\beta), \quad (30)$$

where

$$K(\beta) = \frac{\beta}{1-\beta} + \frac{1-\beta}{\beta} + \beta(1-\beta)$$

is the DGLAP kernel. Gluon self-energy is known to coincide in the axial gauge with a Sudakov form factor whose double logarithmic behavior originates from the product of transverse and longitudinal divergencies. The latter one comes about when one gluon momentum in the loop becomes soft, $\beta \rightarrow 0$. To separate the longitudinal logarithms needed in order to find the function $\omega(p_\perp)$ we keep only the singular piece in $K(\beta)$ and cut the integral in (30) at $\beta = x_\lambda$, using dimensional regularization only for transverse divergency, which turns into

$$\pi_1(x_\lambda, p_\perp^2) = -2N_c \frac{g^2}{8\pi^2} \Gamma\left(1 - \frac{D}{2}\right) [p_\perp^2/4\pi]^{\frac{D}{2}-1} \ln \frac{1}{x_\lambda} = 2\omega(p_\perp) \ln \frac{1}{x_\lambda}. \quad (31)$$

This results into

$$2\omega(p_\perp) = 2N_c \frac{g^2}{8\pi^2} \left[\frac{1}{\epsilon} + \ln \frac{p_\perp^2}{4\pi} + \gamma_E \right] + O(\epsilon),$$

while dimensionally regularized integral (29) gives

$$\begin{aligned} 2\omega(q_\perp) &= N_c \frac{g^2}{8\pi^2} [q_\perp^2/4\pi]^{\frac{D}{2}-1} \Gamma\left(2 - \frac{D}{2}\right) \frac{\Gamma^2(\frac{D}{2}-1)}{\Gamma(D-2)} \\ &= 2N_c \frac{g^2}{8\pi^2} \left[\frac{1}{\epsilon} + \ln \frac{q_\perp^2}{4\pi} + \gamma_E \right] + O(\epsilon). \end{aligned}$$

Thus, although functions $\omega(p_\perp)$ defined by eq.(29) and eq.(31) look different their nonvanishing parts are equal.

Further, if we consider the renormalization of a heavy color source of mass M due to classical current emission, the divergent part of the source self energy amounts to the same expression

$$\pi_M(x_\lambda, p_\perp^2) = -2N_c \frac{g^2}{8\pi^2} \Gamma\left(1 - \frac{n}{2}\right) \left[\frac{M^2 - p^2}{4\pi} \right]^{\frac{n}{2}-1} \ln \frac{1}{x_\lambda}, \quad (32)$$

with source mass M and virtuality $M^2 - p^2 \simeq p_\perp^2$ regardless of an elementary or composite nature of the source.

6 QED cascade

The above treatment can be applied to electrodynamics. Generally speaking, the wave function of the source, which could be any charged object, for instance an electron, is made of the bare source itself surrounded by a cloud of soft photons. The operator Q can probe either the source or photon component of the wave function. If we introduce a cut-off for the longitudinal momentum of the soft photons, we can study the effect of changing this cut-off as we did for the QCD case. We get the same three type of contributions as illustrated in Fig. 5, where the wavy lines now represent photons.

The z -factor is related in this case to the renormalization of the source, and can be obtained from Eq.(32). One needs only to replace g by the electric charge Ze and put $N_c = 1$. The real contribution is only due to the emission off the source line, since photon has no charge. In the corresponding Wilson line (3), the ordering of the fields is not important since they commute at equal light cone time. It is related to the fact that in QED, the soft cascade structure does not assume any additional β ordering (10).

The equation satisfied by the corresponding n_f is the same as Eq.(24), after performing the replacement $g^2 N_c \rightarrow Z^2 e^2$. In the case of photon density, one gets

$$x_\lambda \frac{\partial}{\partial x_\lambda} G_\gamma(x_\lambda, r_\perp, q_\perp) = - \frac{Z^2 e^2}{(2\pi)^3} \frac{\delta^2(q_\perp - r_\perp)}{r_\perp^2} - 2 \frac{Z^2 e^2}{(2\pi)^3} \int \frac{d^2 k_\perp}{(q - k)_\perp^2} G_\gamma(x_\lambda, r_\perp, k_\perp) - 2\omega(q_\perp) G_\gamma(x_\lambda, r_\perp, q_\perp). \quad (33)$$

This equation has just the same form than the BFKL equation (26) for QCD. Let us stress once more that contrarily to the QCD case, $\omega(q_\perp)$ corresponds to the reggeization of the source and not of the photon, which is of different nature. In QCD, in the case where the source is gluon like, $\omega(q_\perp)$ can be associated either to the source or to the cascade. We have used this property to get the expression for $\omega(q_\perp)$. From the point of view of source dressing, it is obtained through the one loop gluon self energy (see Eq.(31)). Equivalently, it is obtained through the normalisation of the wave function (see Eq.(29)).

7 Cascade wave function

Let us now turn back to the amplitude (17). Despite the symbolic character of the infinite ordered product it can be presented in a closed form. Consider to this end the amplitude to emit n quasi-real gluons with momenta $x_n, q_{\perp 1}, \dots, x_1, q_{\perp 1}$, $x_1 > x_2 > \dots > x_n$. It is given by the matrix element

$$\begin{aligned} & \Gamma_{\lambda_n, c_n, \dots, \lambda_1, c_1; \sigma', \sigma; a', a}(x_n, q_{\perp n}, \dots, x_1, q_{\perp 1}; q'_\perp, q_\perp) \\ &= \langle 0 | b_{\sigma', a'}(p_B, q'_\perp) a_{\lambda_n, c_n}(x_n, q_{\perp n}) \cdots a_{\lambda_1, c_1}(x_1, q_{\perp 1}) \Phi^+(x_\lambda) b_{\sigma, a}^+(p_B, q_\perp) | 0 \rangle. \end{aligned} \quad (34)$$

Expanding the brackets, the product in the $\Phi^+(x_\lambda)$ operator can be reorganized as

$$\begin{aligned}\Phi^+(x_\lambda) &= \prod_{x_\lambda}^{x_n} \left[1 - \omega(\hat{P}) \frac{\Delta x}{x} \right] \int_{x_n - \Delta x}^{x_n} d\beta_n \int d^2 k_{\perp n} V^+(\beta_n, k_{\perp n}) \\ &\quad \times \prod_{x_{n-1}}^{x_n} \left[1 - \omega(\hat{P}) \frac{\Delta x}{x} \right] \cdots \prod_{x_1}^{x_2} \left[1 - \omega(\hat{P}) \frac{\Delta x}{x} \right] \\ &\quad \times \int_{x_1 - \Delta x}^{x_1} d\beta_1 \int d^2 k_{\perp 1} V^+(\beta_1, k_{\perp 1}) \prod_{x_1}^1 \left[1 - \omega(\hat{P}) \frac{\Delta x}{x} \right] + \cdots.\end{aligned}$$

The terms which are not explicitly written gives zero contribution to the matrix element (34) when performing the contractions with a operators. Using the fact that

$$\prod_{x_2}^{x_1} \left[1 - \omega(\hat{P}) \frac{\Delta x}{x} \right] = \exp \left\{ - \int_{x_2}^{x_1} \frac{d\beta}{\beta} \omega(\hat{P}) \right\}$$

and that the operator \hat{P} results into the total momentum of the state occuring to the right, we get for the amplitude

$$\begin{aligned}&\Gamma_{\lambda_n, c_n, \dots, \lambda_1, c_1; \sigma', \sigma; a', a}(x_1, q_{\perp 1}, \dots, x_n, q_{\perp n}; q'_\perp, q_\perp) \\ &= \left(\frac{x_\lambda}{x_n} \right)^{\omega(q_{\perp n} + \dots + q_{\perp 1} + q_\perp)} \Gamma_{c_n}^{\lambda_n}(q_{\perp 1}) \left(\frac{x_n}{x_{n-1}} \right)^{\omega(q_{\perp n-1} + \dots + q_{\perp 1} + q_\perp)} \Gamma_{c_{n-1}}^{\lambda_{n-1}}(q_{\perp n-1}) \cdots \\ &\quad \times \Gamma_{c_1}^{\lambda_1}(q_{\perp 1}) \left(\frac{x_1}{1} \right)^{\omega(q_\perp)} \langle 0 | b_{\sigma', a'}(p_B, q'_\perp) b_{\sigma, a}^+(p_B, q_\perp) | 0 \rangle.\end{aligned}$$

This formula, in the LLA approximation, provides all information on the cascade wave function. It resembles multi-reggeon formula but it includes only soft vertex.

8 Conclusion

This paper is based on the ordering of longitudinal variables in soft cascades, which means energy ordering of emitted particles. It differs from collinear approximation which implies angular ordering [15]. In this sense, this treatment describes the dynamics responsible for BFKL evolution, while the collinear one is related to the conventional partonic DGLAP picture. We have shown explicitly that the infrared evolution equation for the parton density in the soft cascade reproduces forward BFKL equation.

In both BFKL and DGLAP equations the evolution is written with respect to a cut-off. DGLAP involves a transverse cut-off Q^2 due to collinear singularities, while in our case the cut-off is longitudinal, due to soft singularities.

In DGLAP case the unintegrated parton distributions, which depend both on the longitudinal and transverse momenta, are related to the derivative of the structure functions with respect to the transverse cut-off. Similarly we have to

consider the derivative with respect to x_λ of the function $G(x_\lambda, r_\perp, p_\perp)$, which satisfies BFKL equation, in order to get the unintegrated distribution.

Our approach is based on a kind of duality in the description of a gluon at high energy. From the viewpoint of the soft vertex the gluon state can be treated as a single particle with given momentum and color. On the other hand the gluon has internal structure, a cloud of many soft gluons surrounding it. It looks like a composite object with a nontrivial wavefunction. Hence a rather simple interpretation of reggeization as a manifestation of soft cascades structure appears in this picture. The trajectory $\omega(q_\perp)$ is nothing more than the gluon Z factor in the axial gauge. Both in QCD and QED, this function is associated with the charged source self energy. In DGLAP case the divergencies occurring at $x \rightarrow 1$ are regulated by the $1/(x-1)_+$ prescription, corresponding to UV Z -factors due to UV renormalization of parton wave functions. In BFKL case the IR divergencies for $k_\perp \rightarrow 0$ disappear because of virtual corrections which can be treated as IR renormalization of the gluon wave function. The equation (26) can be viewed as describing 2 reggeized gluons, or, equivalently, 2 gluon soft cascades.

The extension of this approach for non forward case and for Generalized Leading Log Approximation ([16, 17, 18]) will be presented in a forthcoming paper. It would be also interesting to study more complicated structures like for example the triple Pomeron vertex.

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